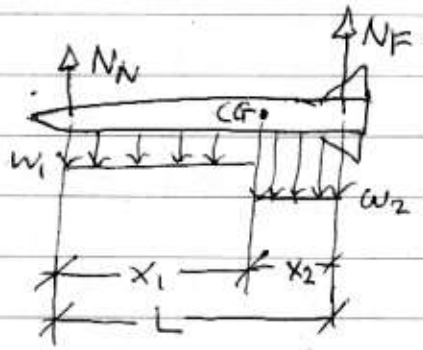
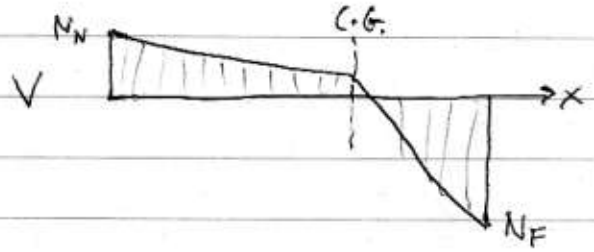
CALCULATE SHEAR & BENDING MOMENT



SHEAR $\quad V(x) = N_N - w_1 x \qquad 0 < x < x_1$

$\qquad\quad V(x) = V_1 - w_2(x - x_1) \qquad x_1 < x < L$

WHERE $V_1 = V_{x_1}$ (AT C.G.)



SHEAR DIAGRAM (TYPICAL)

BENDING MOMENT IS AREA UNDER SHEAR CURVE

FOR $0 < x < x_1$:

$$M(x) = \int (N_N - w_1 x)\, dx$$

$$= \int N_N\, dx - w_1 \int x\, dx$$

$$= N_N x - w_1 \frac{x^2}{2} + C_1 \qquad C_1 = \text{CONSTANT}$$

AT $x = 0$, $M = 0 \Rightarrow C_1 = 0$

$$\boxed{M(x) = N_N x - w_1 \frac{x^2}{2}} \qquad 0 < x < x_1$$

FOR $x_1 < x < L$

$$M(x) = \int [V_1 - w_2 (x - x_1)]\, dx$$

$$= \int V_1\, dx - w_2 \int x\, dx + w_2 \int x_1\, dx$$

$$= V_1 x - w_2 \frac{x^2}{2} + w_2 x_1 x + C_2$$

at $x = L$, $M = 0$

$$\therefore C_2 = w_2 \frac{L^2}{2} - V_1 L - w_2 x_1 L$$

$$C_2 = \frac{w_2 L^2}{2} - L(V_1 + w_2 x_1)$$

$$\Rightarrow M(x) = V_1 x - w_2 \frac{x^2}{2} + w_2 x_1 + w_2 \frac{L^2}{2} - L(V_1 + w_2 x_1)$$

$$\boxed{M(x) = V_1 x + w_2 \left(x_1 x + \frac{L^2}{2} - \frac{x^2}{2}\right) - L(V_1 + w_2 x_1)} \qquad x_1 < x < L$$

FIND LOCATION WHERE $V = 0$

FOR $0 < x < x_1$

$$0 = N_N - w_1 x$$

$$\Rightarrow \boxed{x_{V=0} = \frac{N_N}{w_1}} \quad \text{for } 0 < x < x_1$$

FOR $x_1 < x < L$

$$0 = V_1 - w_2 (x - x_1)$$

$$\boxed{x_{V=0} = \frac{V_1}{w_2} + x_1} \quad \text{for } x_1 < x < L$$

MAXIMUM B.M. OCCURS WHERE $V=0$, OR WHERE SLOPE OF $M(x)$ FUNCTION IS ZERO

$$\frac{dM}{dx} = 0$$

FOR $0 < x < x_1$ (APPLIES ONLY IF $0 < x_{V=0} < x_1$)

$$\frac{dM}{dx} = N_N - \frac{2w_1 x}{2}$$

$$\Rightarrow \quad 0 = N_N - w_1 x$$

$$\boxed{x_{@M_{max}} = \frac{N_N}{w_1}} \qquad 0 < x < x_1$$

← SAME AS FOUND EARLIER AT $V=0$

FOR $x_1 < x < L$

$$\frac{dM}{dx} = V_1 + w_2 x_1 - w_1 x$$

$$\Rightarrow \quad 0 = V_1 + w_2 x_1 - w_1 x$$

$$\boxed{x_{@M_{max}} = \frac{V_1}{w_2} + x_1} \qquad x_1 < x < L$$

← SAME AS FOUND EARLIER AT $V=0$

IF $0 < x_{V=0} < x_1$

$$M(x) = N_N x - w_1 \frac{x^2}{2}$$

AT $x_{V=0}$, $x = \frac{N_N}{w_1}$

$$\Rightarrow \quad M_{max} = N_N \left(\frac{N_N}{w_1}\right) - \frac{w_1}{2}\left(\frac{N_N}{w_1}\right)^2$$

$$= \frac{N_N^2}{w_1} - \frac{N_N^2}{2w_1}$$

$$\boxed{M_{max} = \frac{N_N^2}{2w_1}} \quad \text{FOR } 0 < x_{V=0} < x_1$$

DETERMINE M_{max} IF $X_1 < X_{V=0} < L$

$$X_{V=0} = X_1 + \frac{V_1}{W_2}$$

$$M_{max} = V_1\left(X_1 + \frac{V_1}{W_2}\right) + W_2\frac{L^2}{2} - \frac{W_2}{2}\left(\frac{V_1}{W_2} + X_1\right)^2 - L(V_1 + W_2 X_1)$$

$$= \frac{V_1^2}{W_2} + V_1 X_1 + W_2\left(X_1\frac{V_1}{W_2} + X_1^2\right) + W_2\frac{L^2}{2} - \frac{W_2}{2}\left(\frac{V_1}{W_2} + X_1\right)^2 - LV_1 - LW_2X_1$$

$$= \frac{V_1^2}{W_2} + V_1X_1 + \cancel{W_2}X_1\frac{V_1}{\cancel{W_2}} + W_2X_1^2 + W_2\frac{L^2}{2} - \frac{W_2}{2}\left(\frac{V_1^2}{W_2^2} + 2\frac{V_1}{W_2}X_1 + X_1^2\right) - LV_1 - LW_2X_1$$

$$= \frac{V_1^2}{W_2} + V_1X_1 + V_1X_1 + W_2X_1^2 + W_2\frac{L^2}{2} - \frac{W_2}{2}\frac{V_1^2}{W_2^2} - \cancel{\frac{W_2}{2}}\cancel{2}\frac{V_1}{\cancel{W_2}}X_1 - \frac{W_2}{2}X_1^2 - LV_1 - LW_2X_1$$

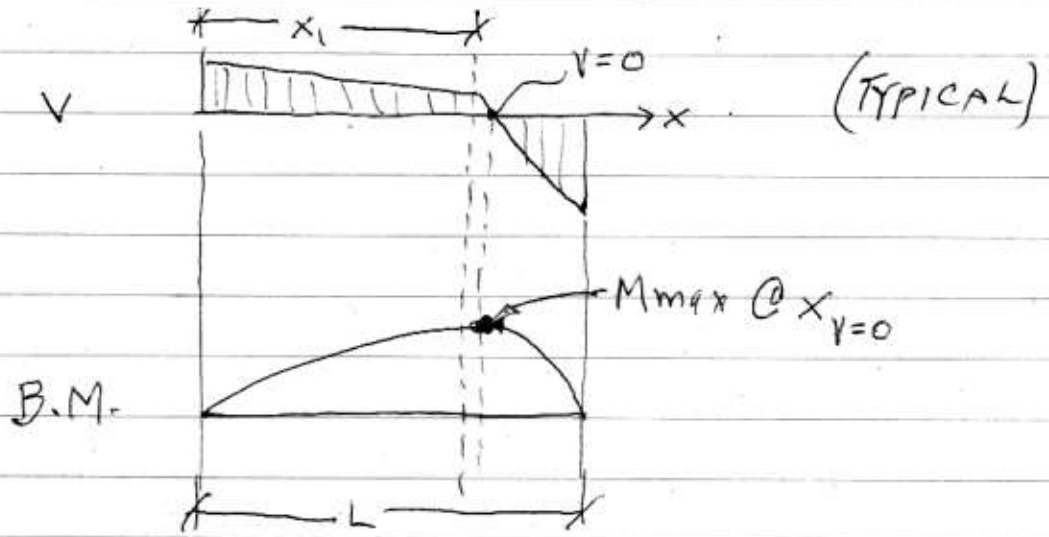
GROUPING LIKE TERMS:

$$= \frac{V_1^2}{2W_2} + V_1X_1 + \frac{W_2}{2}X_1^2 + W_2\frac{L^2}{2} - LV_1 - LW_2X_1$$

$$\Rightarrow \boxed{M_{max} = \frac{V_1^2}{2W_2} - V_1(L - X_1) + W_2\left(\frac{X_1^2}{2} + \frac{L^2}{2} - LX_1\right)}$$

FOR $X_1 < X_{V=0} < L$

SHEAR / BENDING MOMENT DIAGRAM



NOTE : FOR THE VAST MAJORITY OF EX-ROCKETS LOCATION OF ZERO SHEAR & M_{MAX} WILL LIE AFT OF THE C.G.

i.e. IN RANGE

$$x_1 < x < L$$