

DERIVATION OF THRUST EQUATION (3)

$$F = A^* P_0 \sqrt{\frac{2k^2}{k-1} \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{k-1}{k}}\right]} + F_p$$

NEGLECT PRESSURE THRUST TERM, F_p

EQN-2

$$F = \rho^* A^* v^* v_e$$

FROM EQN-7

$$\frac{P_0}{P^*} = \left(1 + \frac{k-1}{2}\right)^{\frac{1}{k-1}} \quad \text{SINCE } M=1 \text{ AT THROAT}$$

FROM EQN-4

$$\frac{T_0}{T^*} = 1 + \frac{k-1}{2} \quad " \quad "$$

FROM EQN-3

$$v^* = \sqrt{k R T^*}$$

AND EQN-12

$$v_e = \sqrt{2 T_0 R \left(\frac{k}{k-1}\right) \eta} \quad \text{WHERE } \eta = \left(1 - \frac{P_e}{P_0}\right)^{\frac{k}{k-1}}$$

THIS GIVES

$$\rho^* = \frac{P_0}{\left(1 + \frac{k-1}{2}\right)^{\frac{1}{k-1}}} = P_0 \left(1 + \frac{k-1}{2}\right)^{\frac{1}{1-k}}$$

FROM IDEAL GAS LAW

$$\rho_0 = \frac{P_0}{RT_0}$$

$$\Rightarrow \rho^* = \frac{P_0}{RT_0} \left(1 + \frac{k-1}{2}\right)^{\frac{1}{1-k}}$$

CRITICAL (THROAT) VELOCITY IS

$$V^* = \sqrt{k R T_0 \frac{1}{1 + \frac{k-1}{2}}}$$

PUT ABOVE INTO EQN. 2

$$F = \frac{P_0}{RT_0} \left(1 + \frac{k-1}{2}\right)^{\frac{1}{1-k}} A^* \sqrt{k R T_0 \frac{1}{1 + \frac{k-1}{2}}} \sqrt{2 T_0 R \left(\frac{k}{k-1}\right) \eta}$$

GATHER TERMS UNDER SQUARE ROOT SIGN:

$$F = \frac{A^* P_0}{RT_0} \left(1 + \frac{k-1}{2}\right)^{\frac{1}{1-k}} \sqrt{2 T_0^2 R^2 k \left(\frac{1}{1 + \frac{k-1}{2}}\right) \left(\frac{k}{k-1}\right) \eta}$$

$$F = \frac{A^* P_0}{\cancel{RT_0}} \left(1 + \frac{k-1}{2}\right)^{\frac{1}{1-k}} \cancel{T_0 R} \sqrt{\frac{2 k^2}{k-1} \left(\frac{1}{1 + \frac{k-1}{2}}\right) \eta}$$

$$F = A^* P_0 \sqrt{\frac{2 k^2}{k-1} \left(1 + \frac{k-1}{2}\right)^{\frac{2}{1-k}} \left(\frac{1}{1 + \frac{k-1}{2}}\right) \eta}$$

(3)

$$F = A^* P_0 \sqrt{\frac{2k^2}{k-1} \left(1 + \frac{k-1}{2}\right)^{\frac{2}{1-k}} \left(1 + \frac{k-1}{2}\right)^{-1} \eta}$$

$$F = A^* P_0 \sqrt{\frac{2k^2}{k-1} \left(1 + \frac{k-1}{2}\right)^{\frac{k+1}{1-k}} \eta}$$

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$$F = A^* P_0 \sqrt{\frac{2k^2}{k-1} \left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}} \left[1 - \left(\frac{P_e}{P_0}\right)^{\frac{k-1}{k}}\right]} \quad \checkmark$$