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2/2023

DERIVE EQUATION FOR CHAMBER PRESSURE FOR TWO-PHASE PRODUCTS

MASS GENERATION RATE:

$$\dot{m}_{gs} = A_b \rho_p r \quad \sim ①$$

RATE AT WHICH PRODUCTS ARE STORED IN COMBUSTION CHAMBER:

$$\frac{d M_{gs}}{dt} = \frac{d}{dt}(\rho_0 v_0) = \rho_0 \frac{dv_0}{dt} + v_0 \frac{d\rho_0}{dt}$$

WHERE ρ_0 = TWO-PHASE GAS DENSITY

$\frac{dv_0}{dt} = A_b r(1-X)$ CHANGE IN CHAMBER VOLUME W.R.T. TIME

$$\Rightarrow \frac{dM_{gs}}{dt} = \rho_0 A_b r(1-X) + v_0 \frac{d\rho_0}{dt} \quad \sim ②$$

WHERE v_0 = FREE VOLUME IN CHAMBER

MASS FLOW RATE THROUGH NOZZLE:

$$\dot{m}_n = \rho^{*''} A^* v^{*''}$$

WHERE $\rho^{*''} = \frac{P_0}{R'' T_0}\left(1 + \frac{k''-1}{2}\right)^{\frac{1}{1-k''}}$

$$V^{*''} = \sqrt{k'' R'' \frac{T_0}{1 + \frac{k''-1}{2}}}$$

THIS GIVES

$$\dot{m}_h = \frac{P_0 A^*}{R'' T_0} \left(1 + \frac{k''-1}{2}\right)^{\frac{1}{1-k''}} \sqrt{k'' R'' \frac{T_0}{1 + \frac{k''-1}{2}}}$$

$$= \frac{P_0 A^*}{R'' T_0} \left(1 + \frac{k''-1}{2}\right)^{\frac{1}{1-k''}} \sqrt{R'' T_0} \sqrt{k'' \left(\frac{1}{1 + \frac{k''-1}{2}}\right)}$$

$$= P_0 A^* \left(\frac{R''^{1/2}}{R''}\right)\left(\frac{T_0^{1/2}}{T_0}\right)\left(1 + \frac{k''-1}{2}\right)^{\frac{1}{1-k''}} \sqrt{k'' \left(\frac{1}{1 + \frac{k''-1}{2}}\right)}$$

$$= \frac{P_0 A^*}{\sqrt{R'' T_0}} \left(1 + \frac{k''-1}{2}\right)^{\frac{1}{1-k''}} \sqrt{k'' \left(\frac{1}{1 + \frac{k''-1}{2}}\right)}$$

$$= \frac{P_0 A^*}{\sqrt{R'' T_0''}} \sqrt{k'' \left(1 + \frac{k''-1}{2}\right)^{\frac{2}{1-k''}} \left(1 + \frac{k''-1}{2}\right)^{-1}}$$

$$= \frac{P_0 A^*}{\sqrt{R'' T_0''}} \sqrt{k'' \left(1 + \frac{k''-1}{2}\right)^{\frac{k''+1}{1-k''}}}$$

$$= \frac{P_0 A^*}{\sqrt{R'' T_0''}} \sqrt{k'' \left(\frac{k''+1}{2}\right)^{\frac{k''+1}{1-k''}}}$$

$$= \frac{P_0 A^*}{\sqrt{R'' T_0''}} \sqrt{k'' \left(\frac{2}{k''+1}\right)^{\frac{k''+1}{k''-1}}}$$

$$\therefore \dot{m}_n = P_0 A^* \sqrt{\frac{k''}{R'' T_0''}} \left(\frac{2}{k''+1}\right)^{\frac{k''+1}{2(k''-1)}} \quad \text{---} (3)$$

THE MASS GENERATION RATE MUST EQUAL THE MASS STORAGE RATE IN CHAMBER PLUS MASS FLOW RATE THROUGH NOZZLE:

$$\dot{m}_{gs} = \frac{dM_{gs}}{dt} + \dot{m}_n$$

FROM (1) AND (2)

$$A_b \rho_p r = \rho_0 A_b r (1-x) + V_0 \frac{d\rho_0}{dt} + \dot{m}_n$$

BURN RATE CAN BE REPRESENTED BY:

$$r = a P_c^n$$

THIS GIVES

$$A_b \rho_p a P_0^n = \rho_0 A_b a P_0^n (1-x) + V_0 \frac{d\rho_0}{dt} + \dot{m}_n$$

$$A_b \rho_p a P_0^n = A_b \rho_0 a P_0^n (1-x) + V_0 \frac{d\rho_0}{dt} + P_0 A^* \sqrt{\frac{k''}{R'' T_0}} \left(\frac{2}{k''+1}\right)^{\frac{k''+1}{2(k''-1)}}$$

FROM IDEAL GAS LAW:

$$\frac{d\rho_0}{dt} = \frac{1}{R'' T_0''} \frac{dP_0}{dt}$$

GIVING

$$A_b \rho_p a P_0^n - A_b \rho_0 a P_0^n (1-x) = \frac{V_0}{R'' T_0''} \frac{dP_0}{dt} + P_0 A^* \sqrt{\frac{k''}{R'' T_0''}} \left(\frac{2}{k''+1}\right)^{\frac{k''+1}{2(k''-1)}}$$

REARRANGE

$$\frac{V_0}{R'' T_0''} \cancel{\frac{dP_0}{dt}}^{\to 0} = A_b a P_0^n \left[\rho_p - \cancel{\rho_0 (1-x)}^{\approx 0}\right] - P_0 A^* \sqrt{\frac{k''}{R'' T_0''}} \left(\frac{2}{k''+1}\right)^{\frac{k''+1}{2(k''-1)}}$$

FOR STEADY-STATE CONDITION, L.H.S. VANISHES
ALSO, ρ_0 IS SMALL IN COMPARISON TO ρ_p

REARRANGE IN TERMS OF P_0

$$\frac{P_0}{P_0^n} = \frac{A_b}{A^*} \frac{a \rho_p}{\sqrt{\frac{k''}{R'' T_0} \left(\frac{2}{k''+1}\right)^{\frac{k''+1}{k''-1}}}}$$

$$\Rightarrow P_0 = \left(\frac{\dot{A}_b}{A^*} \frac{a \rho_p}{\sqrt{\frac{k''}{R'' T_0''} \left(\frac{2}{k''+1} \right)^{\frac{k''+1}{k''-1}}}} \right)^{\frac{1}{1-n}}$$

THIS IS IDENTICAL TO GAS-ONLY EXPRESSION FOR CHAMBER PRESSURE (EQN. 11, CHAMBER PRESSURE WEB PAGE) EXCEPT TWO-PHASE TERMS FOR k, R & T_0