**Worked Example #8**

**Calculate the ideal <u>Chamber Pressure</u>** for a rocket motor with Kn = 350. The nozzle exit pressure is 1 atmosphere (optimum expansion). The propellant is KNSB made with fine-grind oxidizer.

The equation for ideal chamber pressure is

$$P_o = \left[ \frac{A_b}{A^*} \frac{a\,\rho_p}{\sqrt{\frac{k}{R\,T_o}\left(\frac{2}{k+1}\right)^{\frac{k+1}{(k-1)}}}} \right]^{\frac{1}{(1-n)}} \qquad \text{equation 11}$$

From Technical Notepad #3 (http://www.nakka-rocketry.net/techs2.html), KNSB has the following properties:

$k$ = 1.136 *for mixture*

To = 1600 K.

M = 39.86 kg/kmol

The universal gas constant, R' = 8314 N-m/kmol-K

Noting that R = R'/M giving R = 208.6 N-m/kg-K

The propellant density ($\rho_p$) is obtained from the KNSU Chemistry web page: (http://www.nakka-rocketry.net/sorbchem.html )
$\rho_p$ = 1.841 gram/cm$^3$

The burn rate parameters for KNSB are taken from Table 2 of the *KN-Dextrose & KN-Sorbitol Propellants Burn rate Experimentation* web page (http://www.nakka-rocketry.net/bntest.html )

| KN-Sorbitol | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| Pressure range psia | | | a in/sec, (psia) | n | Pressure range Mpa | | | a mm/sec, (Mpa) | n |
| 15 | to | 117 | 0.019 | 0.625 | 0.103 | to | 0.807 | 10.71 | 0.625 |
| 117 | to | 218 | 1.648 | -0.314 | 0.807 | to | 1.50 | 8.763 | -0.314 |
| 218 | to | 550 | 0.330 | -0.013 | 1.50 | to | 3.79 | 7.852 | -0.013 |
| 550 | to | 1020 | 0.011 | 0.535 | 3.79 | to | 7.03 | 3.907 | 0.535 |
| 1020 | to | 1548 | 0.277 | 0.064 | 7.03 | to | 10.67 | 9.653 | 0.064 |

The burn rate coefficient, *a*, and the pressure exponent, *n*, both <u>vary with chamber pressure</u>. As we do not know the chamber pressure value, as that is what we wish to calculate, we simply make an educated "guess" as to which range the chamber pressure will fall in. Assuming that the pressure range will be 3.79 to 7.03 MPa (550 to 1020 psi), the values for our first try will be:

Burn rate coefficient, $a$ = 3.907 mm/(sec-MPa$^n$)

Burn rate pressure exponent, $n$ = 0.535

As the equation for chamber pressure is rather cumbersome, the suggested first step is to simplify the calculation by calculating the terms involving $k$ and $n$

$$\frac{1}{1-n} = \frac{1}{1-0.535} = 2.151$$

$$\frac{2}{k+1} = \frac{2}{1.136+1} = 0.9363$$

$$\frac{k+1}{k-1} = \frac{1.136+1}{1.136-1} = 15.71$$

Since we wish to express the chamber pressure in SI units, we must use consistent units. We will use *m-k-s* (metre-kilogram-second) units for all parameters:

$\rho_P = 1.841 \times 1000 = 1841$ kg/m$^3$

The pressure exponent, $n$, is dimensionless and can be used as is. The burn rate coefficient, $a$, must be converted to *m-k-s* units:

$$a = 3.907 \times \frac{1}{1000} \times \frac{1}{(1\times10^6)^{0.535}} = 2.409 \times 10^{-6}\ \frac{m}{sec}\ \frac{1}{Pa^n}$$

*Note 1*: *1 Pa (Pascal) = 1 N/m$^2$*
*1 MPa = 1×10$^6$ Pa*
*1 m = 1000 mm*

*Note 2*: *for more details on conversion method, see* http://www.nakka-rocketry.net/articles/conversion_a.gif )

The ideal chamber pressure may now be calculated based upon our assumed burn rate parameters:

$$Po = \left[350\frac{2.409\times10^{-6}(1841)}{\sqrt{\frac{1.136}{208.6(1600)}(0.9363)^{15.71}}}\right]^{2.151} = 5951425\ Pa = \mathbf{\underline{5.95\ MPa}}$$

As this value is within the range of assumed chamber pressures, the correct burn rate parameters were used.

To convert to *psi* we divide *Pascals* by 6895 giving Po = **863 psi**

It is wise to check units for consistency:

$$\left(\frac{m}{sec}\frac{m^{2n}}{N^n}\frac{kg}{m^3}\frac{m}{sec}\right)^{\frac{1}{1-n}}$$

Collecting *m* and *sec* terms:

$$\left(\frac{m^{2n-1}}{sec^2}\frac{kg}{N^n}\right)^{\frac{1}{1-n}}$$

Since $kg = N\ sec^2/m$

$$(m^{2n-2}\ N^{1-n})^{\frac{1}{1-n}}$$

Simplifying gives units of ***N/m²***, which is correct.