Worked Example #7

Calculate the ideal <u>Chamber Pressure</u> for a rocket motor with Kn = 180. The nozzle exit pressure is 1 atmosphere (optimum expansion). The propellant is KNSU.

The equation for ideal chamber pressure is

$$P_o = \left[\frac{A_b}{A^*} \frac{a \rho_p}{\sqrt{\frac{k}{R T_o} \left(\frac{2}{k+1} \right)^{\frac{k+1}{(k-1)}}}} \right]^{\frac{1}{(1-n)}} \qquad \text{equation 11}$$

From Technical Notepad #3 (http://www.nakka-rocketry.net/techs.html), KNSU has the following properties:

$k = 1.133$ *for mixture*

To = 1720 K.

M = 41.98 kg/kmol

The universal gas constant, R' = 8314 N-m/kmol-K

Noting that R = R'/M giving R = 198 N-m/kg-K

The propellant density (ρ_p) and burn rate parameters (a, n) are obtained from the KNSU Chemistry web page:
(http://www.nakka-rocketry.net/succhem.html)
$\rho_p = 1.89$ gram/cm^3

Burn rate coefficient, $a = 0.0665$ in/(sec-psi^n)
Burn rate pressure exponent, $n = 0.319$

As the equation for chamber pressure is rather cumbersome, the suggested first step is to simplify the calculation by calculating the terms involving k and n

$$\frac{1}{1-n} = \frac{1}{1-0.319} = 1.468$$

$$\frac{2}{k+1} = \frac{2}{1.133+1} = 0.9376$$

$$\frac{k+1}{k-1} = \frac{1.133+1}{1.133-1} = 16.04$$

Since we wish to express the chamber pressure in SI units, we must use consistent units. We will use *m-k-s* (metre-kilogram-second) units for all parameters:

ρ_p = 1.89 × 1000 = 1890 kg/m^3

The pressure exponent, *n*, is dimensionless and can be used as is. The pressure coefficient, *a*, must be converted to SI units:

$$a = 0.0665 \times \frac{1}{39.37} \times \frac{1}{(6895)^{0.319}} = 0.000101 \ \frac{m}{sec} \frac{1}{Pa^n}$$

Note 1: *1 Pa (Pascal) = 1 N/m^2*
1 m = 39.37 inches
1 psi = 6895 Pa
Note 2: for more details on conversion method, see http://www.nakka-rocketry.net/articles/conversion_a.gif)

The ideal chamber pressure may now be calculated:

$$Po = \left[180 \frac{0.000101(1890)}{\sqrt{\frac{1.133}{198(1720)}}(0.9376)^{16.04}}\right]^{1.468} = 4029044 \ Pa = \underline{\mathbf{4.03\ MPa}}$$

To convert to *psi* we divide *Pascals* by 6895 giving Po = **584 psi**

It is wise to check units for consistency:

$$\left(\frac{m}{sec}\frac{m^{2n}}{N^n}\frac{kg}{m^3}\frac{m}{sec}\right)^{\frac{1}{1-n}}$$

Collecting *m* and *sec* terms:

$$\left(\frac{m^{2n-1}}{sec^2}\frac{kg}{N^n}\right)^{\frac{1}{1-n}}$$

Since *kg = N sec^2 /m*

$$(m^{2n-2}\ N^{1-n})^{\frac{1}{1-n}}$$

Simplifying gives units of ***N/m^2***, which is correct.