Worked Example #5

Calculate the ideal <u>Thrust</u> and <u>Thrust Coefficient</u> for a rocket motor operating at 68 atmospheres chamber pressure and exhausts to ambient. The nozzle has a throat diameter of 10mm and has an exit diameter of 20mm. The nozzle exit pressure is 4 atmospheres. The propellant is KNSB.

Po = Stagnation pressure (chamber pressure), 68 atmospheres

Pa = Ambient pressure, 1 atmosphere

From Technical Notepad #3 (http://www.nakka-rocketry.net/techs2.html), KNSB has the following properties:

k = 1.04 *2-phase flow*

The equation for ideal thrust is

$$F = A^* Po \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{k-1}{k}}\right]} + (P_e - P_a)A_e \qquad \text{equation 3}$$

As this is a rather cumbersome equation, the suggested first step is to simplify the calculation by calculating the terms involving "k"

$$\frac{k^2}{k-1} = \frac{(1.04)^2}{1.04-1} = 27.04$$

$$\frac{2}{k+1} = \frac{2}{1.04+1} = 0.9804$$

$$\frac{k+1}{k-1} = \frac{1.04+1}{1.04-1} = 51.0$$

$$\frac{k-1}{k} = \frac{1.04-1}{1.04} = 0.0385$$

The pressure ratio is likewise calculated

$$\frac{Pe}{Po} = \frac{4}{68} = 0.0588$$

The throat cross-sectional area, A*, is

$$A^* = \frac{\pi}{4}(10.0)^2 = 78.5mm^2$$

The nozzle exit cross-sectional area, Ae, is

$$A^* = \frac{\pi}{4}(20.0)^2 = 314mm^2$$

Since we wish to express the thrust in terms of Newtons (or pounds-force), we must use consistent units. To obtain thrust in Newtons, we use *m-k-s* (metre-kilogram-second) units for all parameters:

$A^* = 78.5 /1000^2 = 78.5\text{x}10^{-6}$ m^2

$Ae = 314 /1000^2 = 314\text{x}10^{-6}$ m^2

Likewise, pressure is converted to N/m^2

$$Po = 68\ atm\ \times 101325\ \frac{N/_{m^2}}{atm} = 6.89\ \times 10^6\ \frac{N}{m^2}$$

$$Pe = 4\ atm\ \times 101325\ \frac{N/_{m^2}}{atm} = 0.405\ \times 10^6\ \frac{N}{m^2}$$

$Pa = 101325$ N/m^2

The ideal thrust may now be calculated. Out of interest, we will first calculate the momentum thrust (F`) and the pressure thrust (F``) terms separately:

$$F' = 78.5 \times 10^{-6}(6.89 \times 10^6)\sqrt{2(27.04)(0.9804)^{51}(1-(0.0588)^{0.0385})} = 771\ N.$$

$$F" = (0.405 \times 10^6 - 101325)\ 314 \times 10^{-6} = 95.4 \text{ N.}$$

(to convert to "pounds force", we divide Newtons by 4.448, giving F' = 173 lbf and F" = 21.4 lbf)

Providing a total ideal thrust of **866 N**. (195 lbf).

It is important to always check units for consistency:

$$F = m^2 \frac{N}{m^2}\sqrt{dimensionless} + \left(\frac{N}{m^2} - \frac{N}{m^2}\right) m^2$$

Therefore

$$F = \cancel{m}^2 \frac{N}{\cancel{m}^2}\sqrt{dimensionless} + \left(\frac{N}{\cancel{m}^2} - \frac{N}{\cancel{m}^2}\right)\cancel{m}^2$$

Units are confirmed to be correct (N.)

The equation for Thrust Coefficient is:

$$C_f = \sqrt{\frac{2k^2}{k-1}\left(\frac{2}{k+1}\right)^{\frac{k+1}{k-1}}\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{k-1}{k}}\right]} + \frac{(P_e - P_a)A_e}{P_o \; A^*} \qquad \text{equation 5}$$

Note that the term inside the square root sign is identical to that of the thrust equation.

Out of interest, we will first calculate the momentum Thrust Coefficient and the pressure Thrust Coefficient terms separately.

$$C_f{}' = \sqrt{2(27.04)(0.9804)^{51}(1-(0.0588)^{0.0385})} = 1.43$$

$$C_f{}'' = \frac{(0.405\times10^6 - 101325)\; 314\times10^{-6}}{6.89x10^6\;(78.5x10^{-6})} = 0.18$$

Giving a ideal total Thrust Coefficient of:

Cf = 1.61

A check of the units will confirm that Cf is dimensionless as expected.