Worked Example #4

Calculate the nozzle flow exit velocity for a rocket motor operating at 68 atmospheres chamber pressure, expanding to ambient air. The propellant is sorbitol based KNSB. Neglect losses such as that attributed to two-phase flow and combustion inefficiency.

Po = Stagnation pressure (chamber pressure), 68 atmospheres

Pe = Pressure at nozzle exit plane, 1 atmosphere

From Technical Notepad #3 (http://www.nakka-rocketry.net/techs2.html), KNSB has the following properties:

k = 1.04

To = 1600 K.

M = 39.86 kg/kmol

The universal gas constant, R' = 8314 N-m/kmol-K

The equation that determines nozzle exit velocity is

$$V_e = \sqrt{2\,T_o\left(\frac{R'}{M}\right)\left(\frac{k}{k-1}\right)\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{k-1}{k}}\right]} \qquad \text{equation 12}$$

As this is a rather cumbersome equation, the suggested first step is to simplify the calculation by calculating the terms involving "k"

$$\frac{k}{k-1} = \frac{1.04}{1.04-1} = 26.0$$

$$\frac{k-1}{k} = \frac{1.04-1}{1.04} = 0.0385$$

The pressure ratio is likewise calculated

$$\frac{Pe}{Po} = \frac{1}{68} = 0.0147$$

The ratio R'/M is also calculated

$$\frac{R'}{M} = \frac{8314}{39.86} = 208.58$$

The nozzle flow exit velocity is now calculated

$$v_e = \sqrt{2(1600)208.58(26.0)[1-(0.0147)^{0.0385}]} = 1612\ metres/second$$

To convert to “feet per second” multiply by 3.281, giving ve = 5289 feet/second

It is important to always check units for consistency:

$$v_e = \sqrt{K\ \frac{Nm}{kmol\ K}\frac{kmol}{kg}}$$

Recall (from F=m a) that a Newton is equal to a kilogram-meter per second squared

$$\frac{kg\ m}{s^2}$$

Therefore

$$v_e = \sqrt{K\ \frac{kg\ m\ m}{kmol\ K\ s^2}\frac{kmol}{kg}} = \frac{m}{s}$$

Units are correct.