Worked Example #4

Calculate the nozzle flow exit velocity for a rocket motor operating at 68 atmospheres chamber pressure, expanding to ambient air. The propellant is sorbitol based KNSB. Neglect losses such as that attributed to two-phase flow and combustion inefficiency.

Po = Stagnation pressure (chamber pressure), 68 atmospheres
Pe = Pressure at nozzle exit plane, 1 atmosphere

From Technical Notepad #3 (http://www.nakka-rocketry.net/techs2.html), KNSB has the following properties:
k = 1.04
To = 1600 K.
M = 39.86 kg/kmol

The universal gas constant, $R' = 8314 \text{ N-m/kmol-K}$

The equation that determines nozzle exit velocity is

$$v_e = \sqrt{\frac{2 T_o \left( \frac{R'}{M} \right) \left( \frac{k}{k-1} \right) \left[ 1 - \left( \frac{P_e}{P_o} \right) \left( \frac{k-1}{k} \right) \right]} \quad \text{equation } 12$$

As this is a rather cumbersome equation, the suggested first step is to simplify the calculation by calculating the terms involving “k”

$$\frac{k}{k-1} = \frac{1.04}{1.04 - 1} = 26.0$$

$$\frac{k-1}{k} = \frac{1.04 - 1}{1.04} = 0.0385$$

The pressure ratio is likewise calculated

$$\frac{P_e}{P_o} = \frac{1}{68} = 0.0147$$

The ratio $R'/M$ is also calculated

$$\frac{R'}{M} = \frac{8314}{39.86} = 208.58$$

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The nozzle flow exit velocity is now calculated

\[ v_e = \sqrt{2(1600)208.58(26.0)[1 - (0.0147)^{0.0385}]} = 1612 \text{ metres/second} \]

To convert to “feet per second” multiply by 3.281, giving \( v_e = 5289 \text{ feet/second} \)

It is important to always check units for consistency:

\[ v_e = \sqrt{\frac{K Nm}{kmol K kg}} \]

Recall (from \( F=ma \)) that a Newton is equal to a kilogram-meter per second squared

\[ \frac{kg \ m}{s^2} \]

Therefore

\[ v_e = \sqrt{\frac{K kg \ m \ m}{kmol K s^2 \ kg}} = \frac{m}{s} \]

Units are correct.