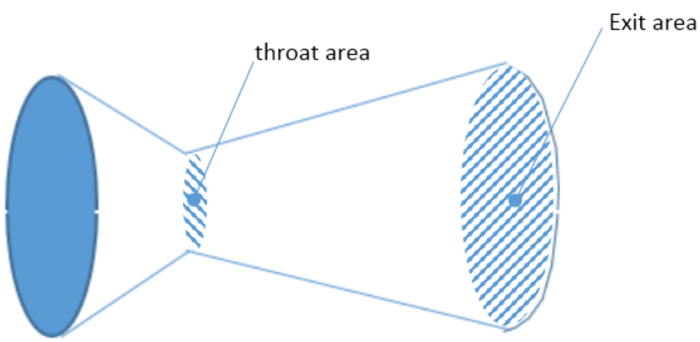
Worked Example #3

Calculate the nozzle Optimum Expansion ratio for a rocket motor operating at 64 atmospheres chamber pressure, expanding to ambient air. Also calculate the nozzle cone exit diameter for a throat diameter of 10 mm.

Po = Stagnation pressure (chamber pressure), atmospheres

Pe = Pressure at nozzle exit plane, atmospheres



For this example, we'll assume k = 1.15

The equation that determines optimum expansion ratio is

$$\frac{A^*}{A_e} = \left(\frac{k+1}{2}\right)^{\frac{1}{k-1}} \left(\frac{P_e}{P_o}\right)^{\frac{1}{k}} \sqrt{\left(\frac{k+1}{k-1}\right)\left[1-\left(\frac{P_e}{P_o}\right)^{\frac{k-1}{k}}\right]}$$

As this is a rather cumbersome equation, the suggested first step is to simplify the calculation by calculating the terms involving "k"

$$\frac{k+1}{2} = \frac{1.15+1}{2} = 1.075$$

$$\frac{1}{k-1} = \frac{1}{1.15-1} = 6.667$$

$$\frac{1}{k} = \frac{1}{1.15} = 0.870$$

$$\frac{k+1}{k-1} = \frac{1.15+1}{1.15-1} = 14.333$$

$$\frac{k-1}{k} = \frac{1.15-1}{1.15} = 0.130$$

The pressure ratio is likewise calculated

$$\frac{Pe}{Po} = \frac{1}{64} = 0.0156$$

The area ratio is next calculated

$$\frac{A*}{Ae} = (1.075)^{6.667}(0.0156)^{0.870}\sqrt{14.333[1-(0.0156)^{0.130}]} = 0.1066$$

The Optimum Expansion Ratio is the reciprocal of this value

$$\frac{Ae}{A*} = \frac{1}{0.1066} = 9.37$$

Note that these ratios are dimensionless.

The nozzle cone exit diameter (De) can now be calculated. Cross-sectional area is related to diameter by the following relationship

$$A = \frac{\pi}{4}D^2$$

Since D*= 10mm,

$$A_* = \frac{\pi}{4}(10)^2 = 78.5mm^2$$

And exit cone diameter is obtained by use of the area ratio and throat diameter:

$$D_e = \sqrt{\frac{4(9.37)78.5}{\pi}} = 30.6mm$$