Worked Example #2

Calculate variation of flow properties temperature, density and pressure with varying Mach number and plot on a graph.

T = Flow temperature at a given Mach number downstream in nozzle

To = Stagnation temperature

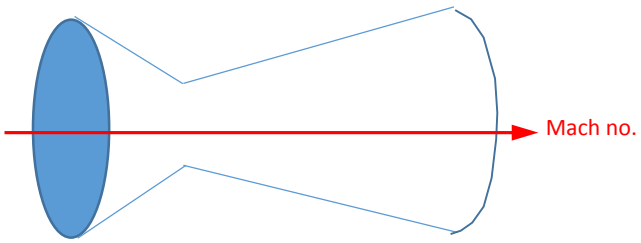
ρ = Flow density at a given Mach number downstream in nozzle

ρo = Stagnation density

P = Flow pressure at a given Mach number downstream in nozzle

Po = Stagnation pressure

M = Mach number of flow at a given downstream location in nozzle



For this example, we'll assume k = 1.15

The equations that relate Temperature, Pressure and Density to Mach number are

$$\frac{T_o}{T} = 1 + \frac{k-1}{2} M^2$$

$$\frac{P_o}{P} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{k}{k-1}}$$

$$\frac{\rho_o}{\rho} = \left(1 + \frac{k-1}{2} M^2\right)^{\frac{1}{k-1}}$$

The suggested first step is to simplify the calculation by calculating the terms involving "k"

$$\frac{k-1}{2} = \frac{1.15-1}{2} = 0.075$$

$$\frac{k}{k-1} = \frac{1.15}{0.15} = 7.667$$

$$\frac{1}{k-1} = \frac{1}{1.15-1} = 6.667$$

To plot the results on a graph, properties for Mach numbers from 0 to 3.00 are calculated, at step sizes of 0.2. By inspection of the equations, it can be seen that when M=1, each of the properties are unity, so start with M=0.2

$$\frac{To}{T} = 1 + 0.075(0.2)^2 = 1.003$$

$$\frac{Po}{P} = [1 + 0.075(0.2)^2]^{7.667} = 1.023$$

$$\frac{\rho o}{\rho} = [1 + 0.075(0.2)^2]^{6.667} = 1.020$$

However, we are interested in the ratio of the downstream property to the stagnation property. So we calculate the reciprocals of each.

$$\frac{T}{To} = \frac{1}{1.003} = 0.997$$

$$\frac{P}{Po} = \frac{1}{1.023} = 0.977$$

$$\frac{\rho}{\rho o} = \frac{1}{1.020} = 0.980$$

Of course, using a spreadsheet program such as Excel makes short work of doing these calculations. Excel is used to generate the graph. The final results and graph are shown below.

Note that both Mach number and property ratio are dimensionless.

M	T/To	ρ/ρo	P/Po
0.00	1.000	1.000	1.000
0.20	0.997	0.980	0.977
0.40	0.988	0.924	0.913
0.60	0.974	0.837	0.815
0.80	0.954	0.732	0.698
1.00	0.930	0.617	0.574
1.20	0.903	0.505	0.456
1.40	0.872	0.401	0.349
1.60	0.839	0.310	0.260
1.80	0.805	0.235	0.189
2.00	0.769	0.174	0.134
2.20	0.734	0.127	0.093
2.40	0.698	0.091	0.064
2.60	0.664	0.065	0.043
2.80	0.630	0.046	0.029
3.00	0.597	0.032	0.019