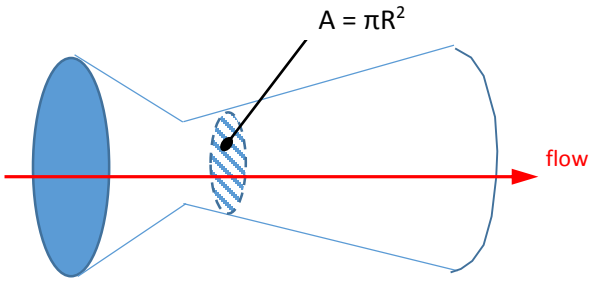
Worked Example #1

Calculate nozzle area ratio (A/A*) with varying Mach number and plot on a graph.

A = cross-sectional area of nozzle passage at a given downstream location in nozzle

A* = cross-sectional area of nozzle throat

M = Mach number of flow at a given downstream location in nozzle



For this example, we'll assume k = 1.15

The equation that relates area ratio to Mach number is

$$\frac{A}{A^*} = \frac{1}{M}\left(\frac{1 + \frac{k-1}{2} M^2}{1 + \frac{k-1}{2}}\right)^{\frac{k+1}{2(k-1)}}$$

The suggested first step is to simplify the calculation by calculating the terms involving "k"

$$\frac{k-1}{2} = \frac{1.15-1}{2} = 0.075$$

$$\frac{k+1}{2(k-1)} = \frac{1.15+1}{2(1.15-1)} = 7.167$$

To plot the results on a graph, area ratios for Mach numbers from 0 to 3.00 are calculated, at step sizes of 0.2.

Note that for M=0, the term 1/M is undefined (cannot divide by zero) so we'll start with M=0.2

$$\frac{A}{A*} = \frac{1}{M}\left(\frac{1 + \frac{k-1}{2} M^2}{1 + \frac{k-1}{2}}\right)^{\frac{k+1}{2(k-1)}}$$

$$\frac{A}{A*} = \frac{1}{0.2}\left(\frac{1+0.075(0.2)^2}{1+0.075}\right)^{7.167} = 3.042$$

Of course, using a spreadsheet program such as Excel makes short work of doing these calculations. Excel is used to generate the graph. The final results and graph are shown below.

Note that both Mach number and area ratio are dimensionless.

M	A/A*
0.00	-
0.20	3.042
0.40	1.622
0.60	1.201
0.80	1.042
1.00	1.000
1.20	1.035
1.40	1.137
1.60	1.310
1.80	1.573
2.00	1.952
2.20	2.491
2.40	3.253
2.60	4.329
2.80	5.850
3.00	8.002